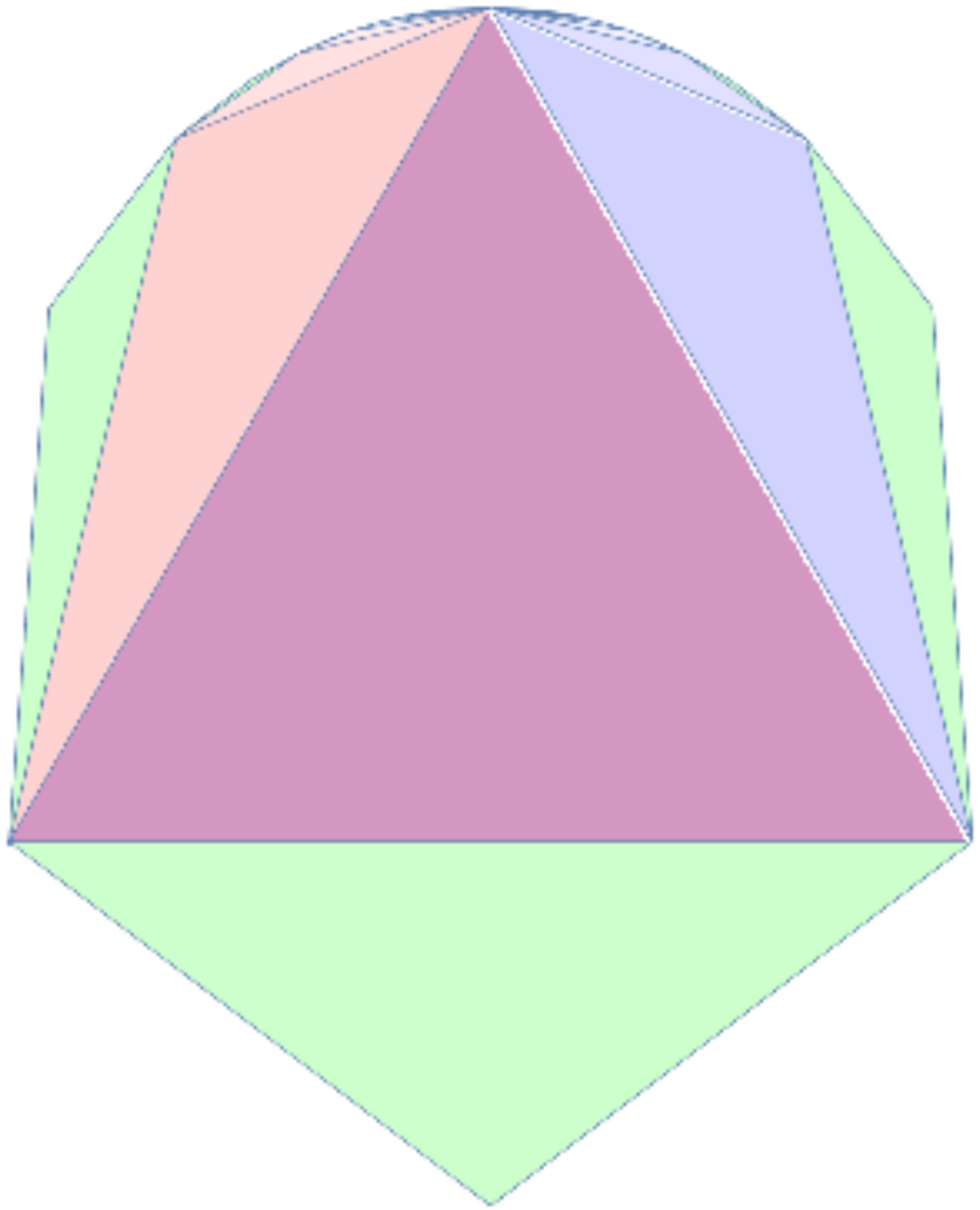
Superpotentials from Singular Divisors



Naomi Gendler
Cornell University
String Phenomenology 2022

Based on:

2204.06566 with Manki Kim, Liam McAllister, Jakob Moritz, and Mike Stillman

Motivation (why care?)

Motivation (why care?)

- Crucial task for type IIB phenomenology: **calculate the superpotential, W.**

[cf. Kachru, Kallosh, Linde, Trivedi'03; Balusubramanian, Berglund, Conlon, Quevedo'05,...]

Motivation (why care?)

- Crucial task for type IIB phenomenology: **calculate the superpotential, W.**
  [cf. Kachru, Kallosh, Linde, Trivedi'03; Balusubramanian, Berglund, Conlon, Quevedo'05,...]
- Witten '96 showed us that Euclidean D3-branes contribute to W if they wrap **smooth, rigid divisors.**

Motivation (why care?)

- Crucial task for type IIB phenomenology: **calculate the superpotential, W.**

  [cf. Kachru, Kallosh, Linde, Trivedi'03; Balusubramanian, Berglund, Conlon, Quevedo'05,...]

- Witten '96 showed us that Euclidean D3-branes contribute to W if they wrap **smooth, rigid divisors.**

- But many (most?) divisors in CY3s **are singular!**

Motivation (why care?)

- Crucial task for type IIB phenomenology: **calculate the superpotential, W.**

[cf. Kachru, Kallosh, Linde, Trivedi'03; Balusubramanian, Berglund, Conlon, Quevedo'05,...]

- Witten '96 showed us that Euclidean D3-branes contribute to W if they wrap **smooth, rigid divisors.**

- But many (most?) divisors in CY3s **are singular!**

When do Euclidean D3-branes wrapped on singular divisors contribute to the superpotential?

Motivation (why care?)

- Crucial task for type IIB phenomenology: **calculate the superpotential, W.**

[cf. Kachru, Kallosh, Linde, Trivedi’03; Balusubramanian, Berglund, Conlon, Quevedo’05,...]

- Witten ’96 showed us that Euclidean D3-branes contribute to W if they wrap **smooth, rigid divisors.**

- But many (most?) divisors in CY3s **are singular!**

When do Euclidean D3-branes wrapped on

singular divisors contribute to the superpotential?

- And a teaser: the answer to this question will lead us to discover **modular superpotentials,** which hint at a new strong-weak duality involving inversions of divisor volumes.

Outline

Outline

1. (Symmetric) flops in Calabi-Yau threefolds

Outline

1. (Symmetric) flops in Calabi-Yau threefolds
2. Superpotential contributions

Outline

1. (Symmetric) flops in Calabi-Yau threefolds
2. Superpotential contributions
3. A modular superpotential

Flops in Calabi-Yaus

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

What can happen?

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

What can happen?

1. An infinite tower of states becomes massless

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

What can happen?

1. An infinite tower of states becomes massless
2. A finite number of states becomes massless and you end up in the moduli space of a new Calabi-Yau, X'.

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

What can happen?

1. An infinite tower of states becomes massless
2. A finite number of states becomes massless and you end up in the moduli space of a new Calabi-Yau, X'.
3. A finite number of states becomes massless and you end up in the moduli space of the same Calabi-Yau, X.

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

What can happen?

1. An infinite tower of states becomes massless
2. A finite number of states becomes massless and you end up in the moduli space of a new Calabi-Yau, X'.
3. A finite number of states becomes massless and you end up in the moduli space of the same Calabi-Yau, X.

"flops"

See talk by Fabian!

Flops in Calabi-Yaus

Consider type IIB string theory on a Calabi-Yau threefold, X.

Now consider shrinking and blowing up a curve class $[\mathscr{C}]$ in X such that

$$[\mathscr{C}] \to -[\mathscr{C}]$$

What can happen?

1. An infinite tower of states becomes massless

"flops"

See talk by Fabian!

2. A finite number of states becomes massless and you end up in the moduli space of a new Calabi-Yau, X'.
3. A finite number of states becomes massless and you end up in the moduli space of the same Calabi-Yau, X.

Note: flops are ubiquitous! [Demirtas, McAllister, Rios-Tascon '20; Brodie, Constantin, Lukas, Ruehle '21]

Flops in Calabi-Yaus

Flops in Calabi-Yaus

Consider a divisor in X that intersects the curve $[\mathscr{C}]$ that we are flopping.

Flops in Calabi-Yaus

Consider a divisor in X that intersects the curve $[\mathscr{C}]$ that we are flopping.

What happens to this divisor as we go through the flop transition?

Flops in Calabi-Yaus

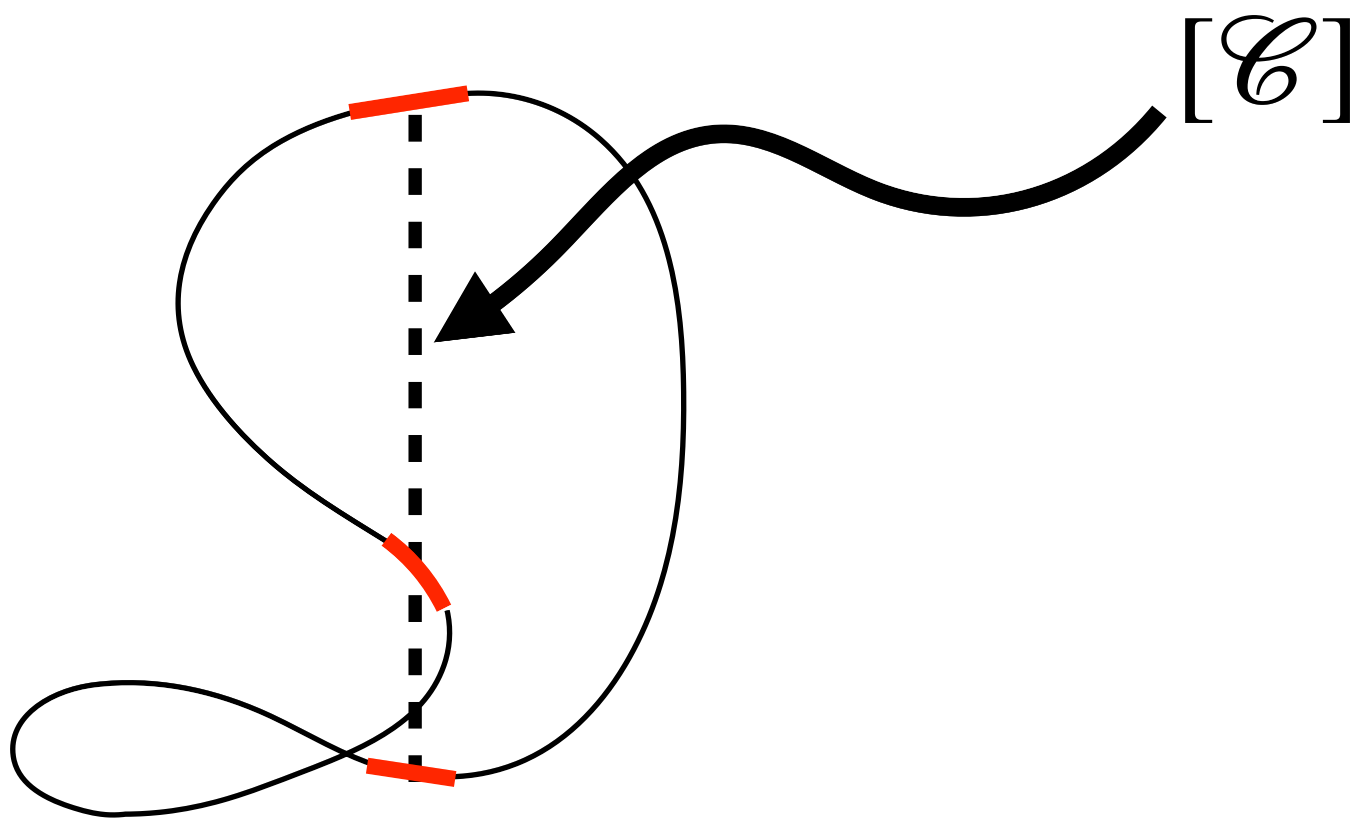
Consider a divisor in X that intersects the curve $[\mathscr{C}]$ that we are flopping.

What happens to this divisor as we go through the flop transition?

Flops in Calabi-Yaus

Consider a divisor in X that intersects the curve $[\mathscr{C}]$ that we are flopping.

What happens to this divisor as we go through the flop transition?

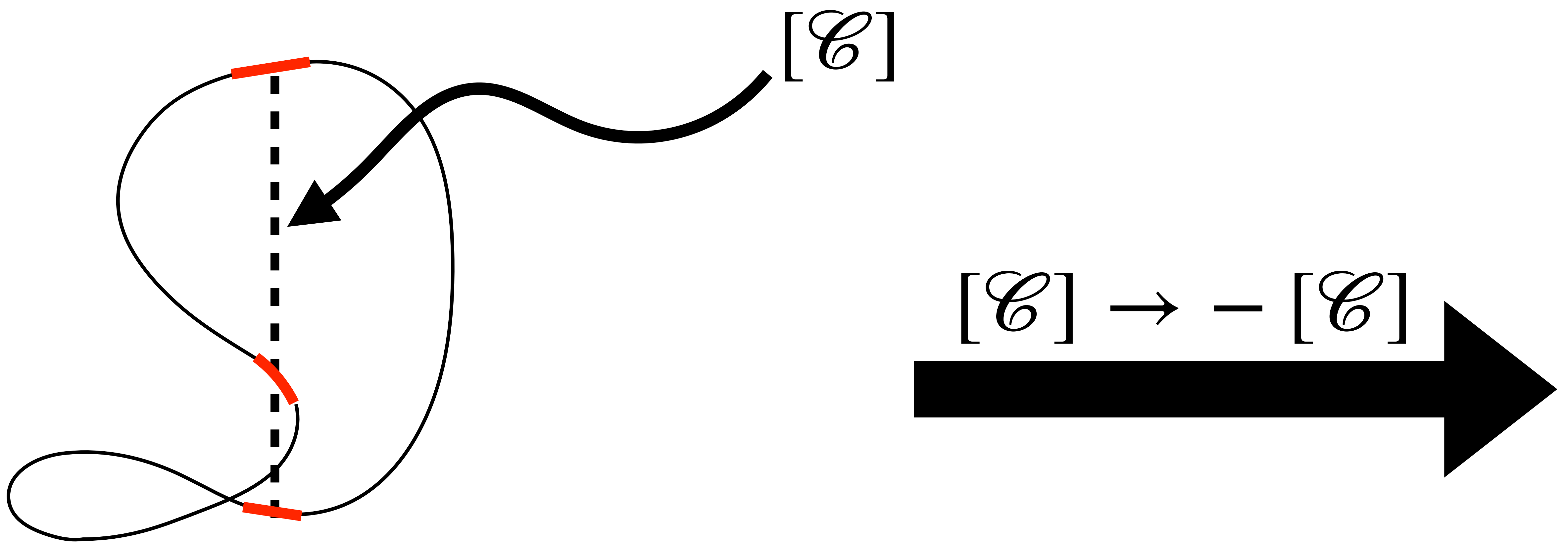


A divisor D in X.

Flops in Calabi-Yaus

Consider a divisor in X that intersects the curve $[\mathscr{C}]$ that we are flopping.

What happens to this divisor as we go through the flop transition?

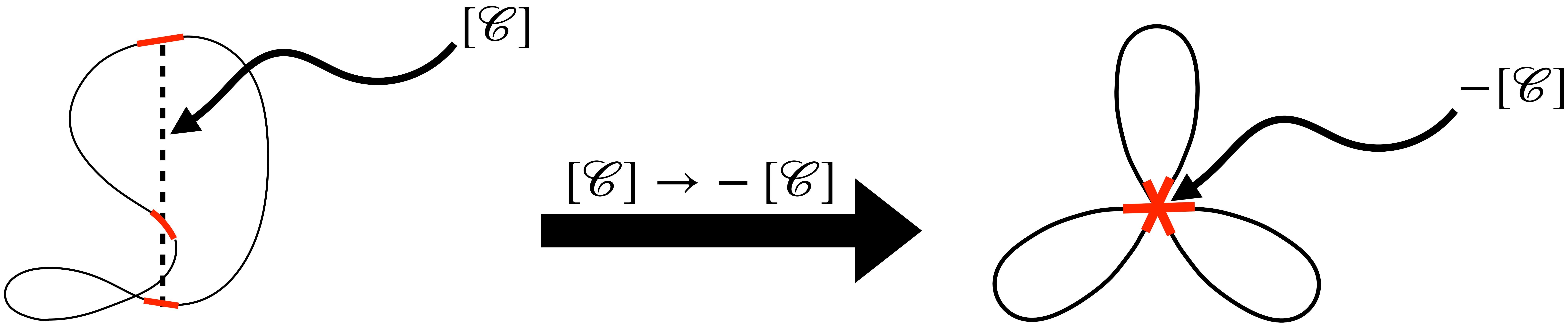


A divisor D in X.

Flops in Calabi-Yaus

Consider a divisor in X that intersects the curve $[\mathscr{C}]$ that we are flopping.

What happens to this divisor as we go through the flop transition?

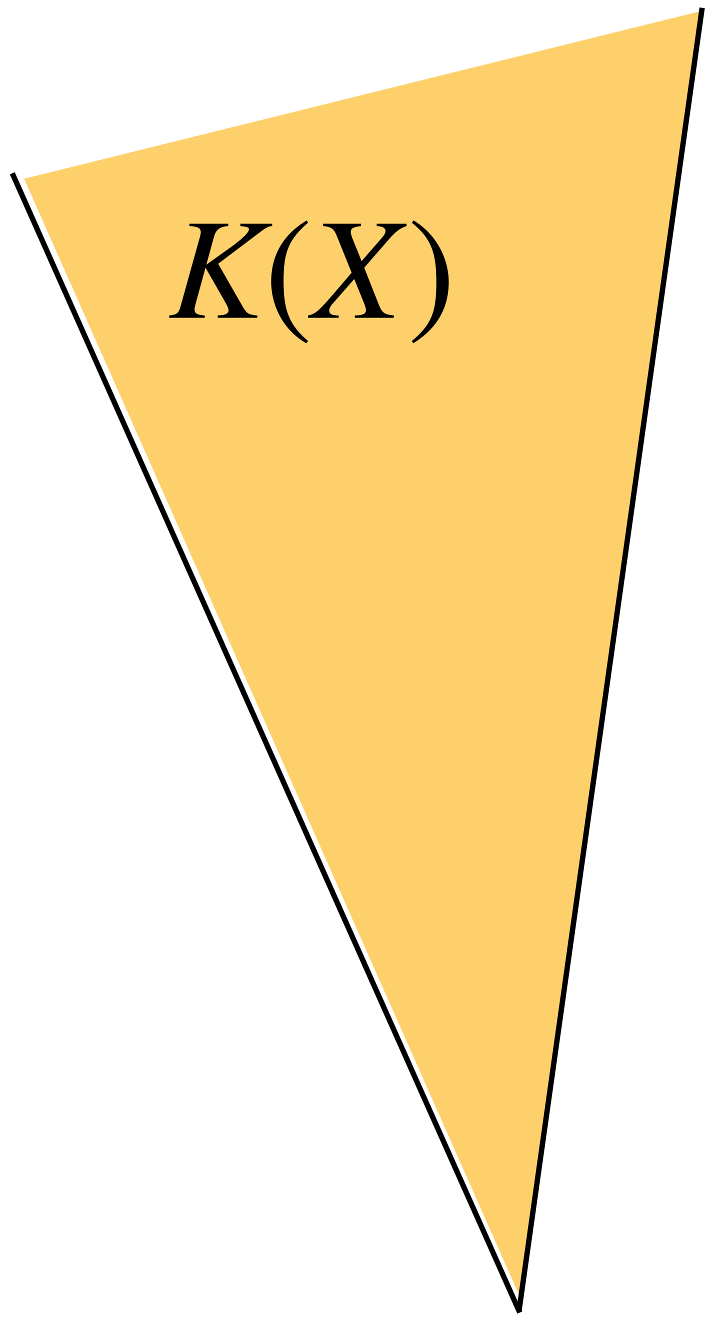


A divisor D in X.

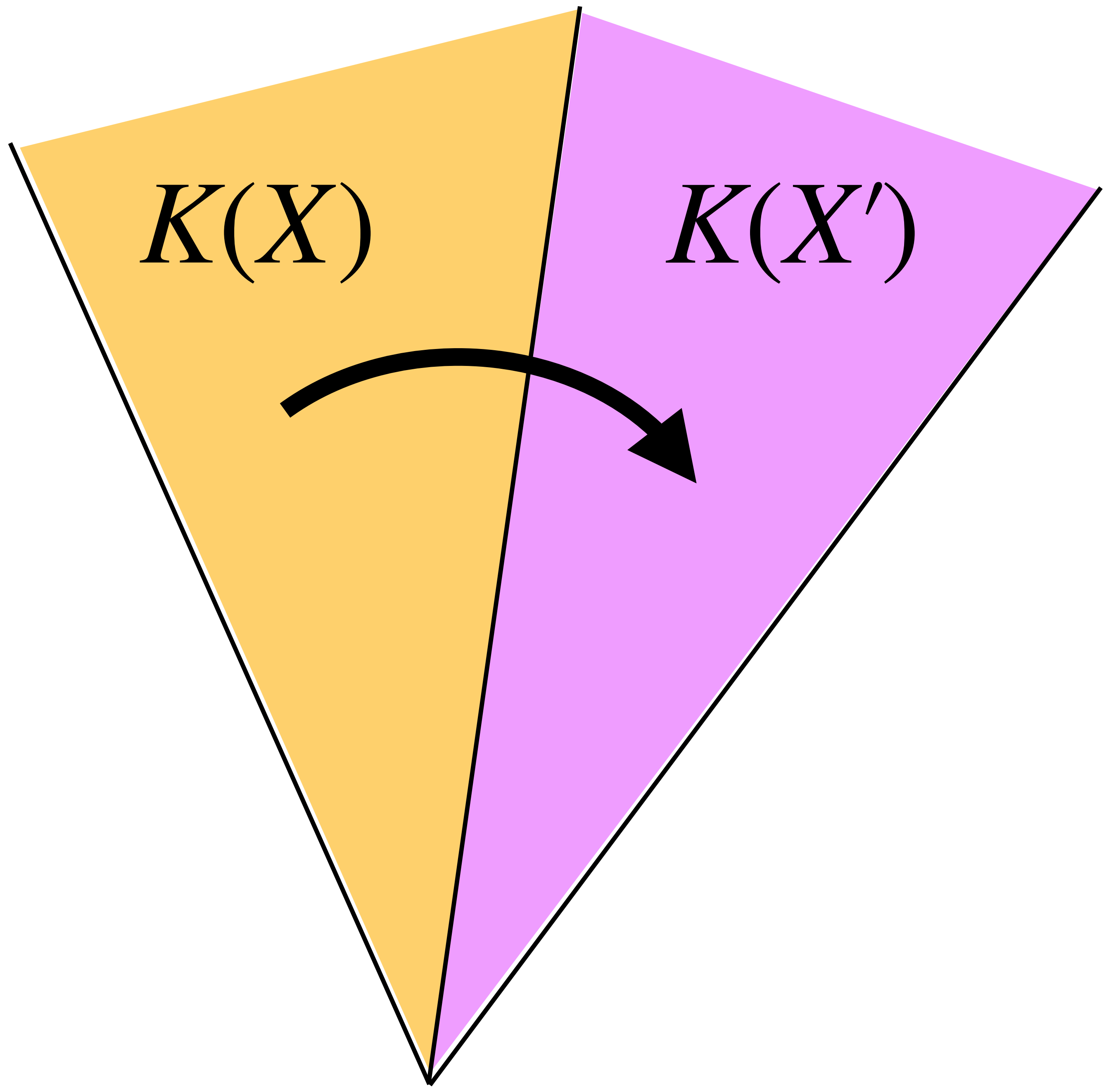
A divisor D' in X'.

Symmetric Flops in Calabi-Yaus

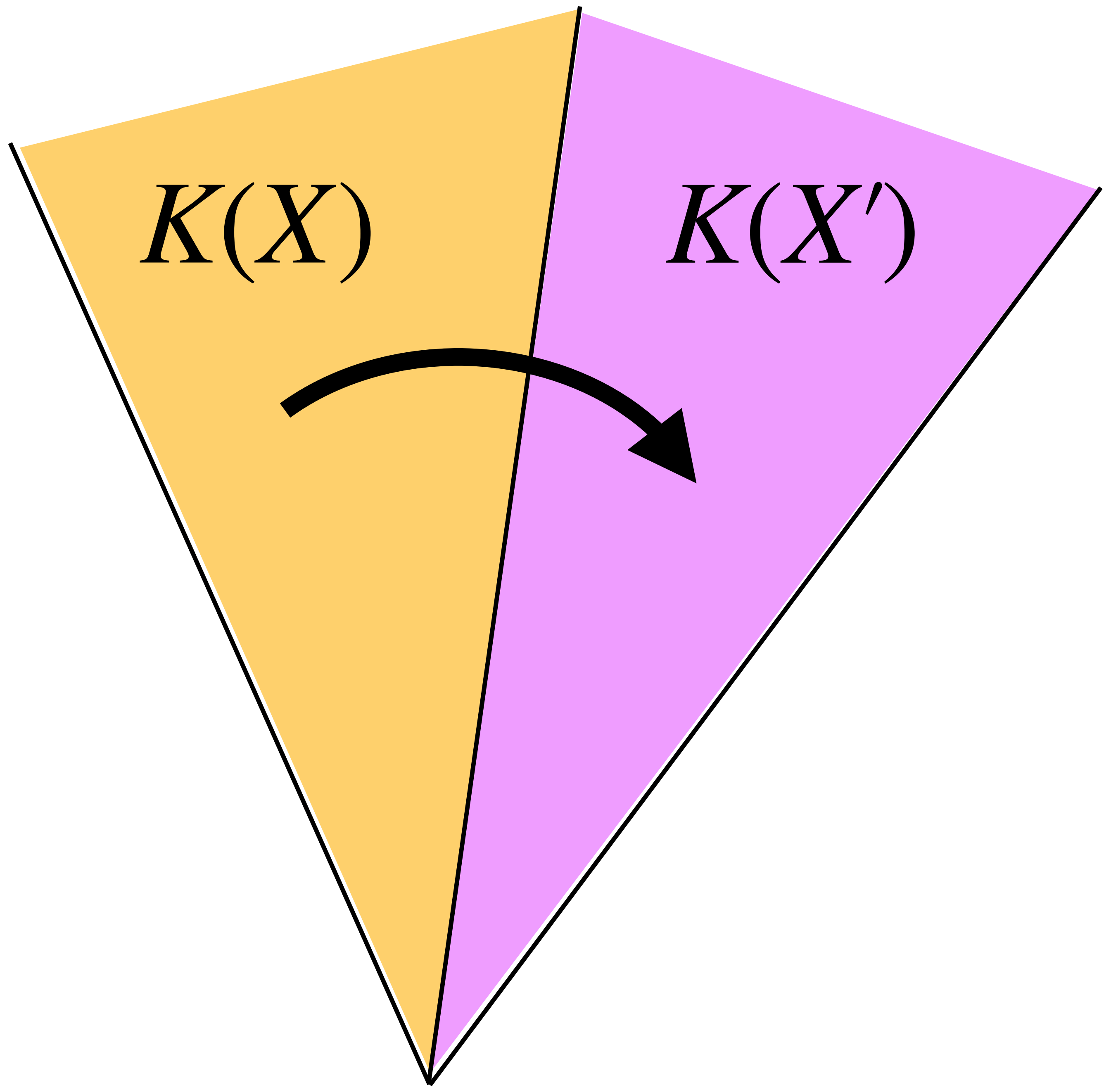
Symmetric Flops in Calabi-Yaus



Symmetric Flops in Calabi-Yaus



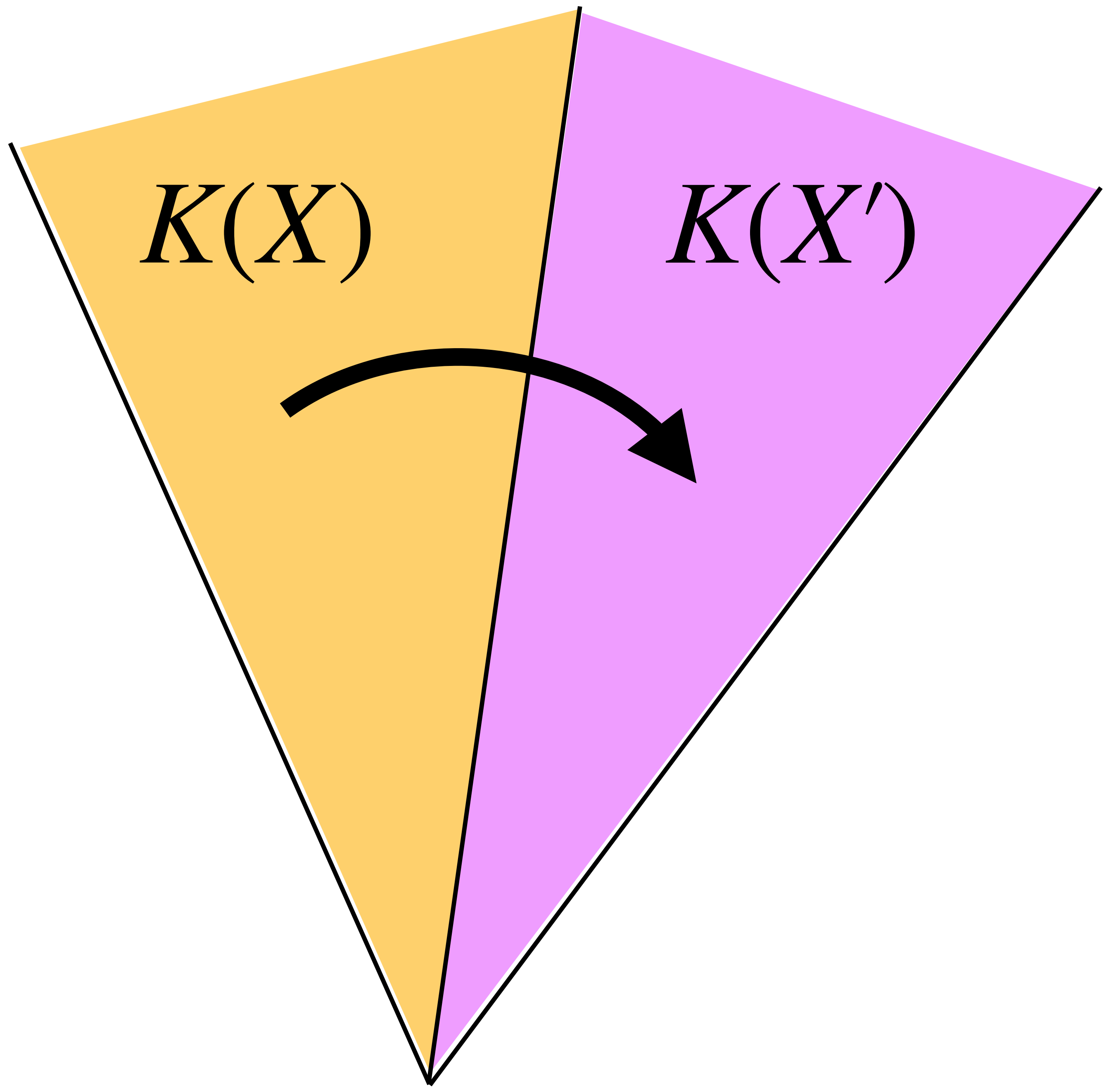
Symmetric Flops in Calabi-Yaus



Symmetric flop: $X \cong X'$

[Brodie, Constantin, Lukas ’20; Brodie, Constantin, Lukas, Ruehle ’21]

Symmetric Flops in Calabi-Yaus



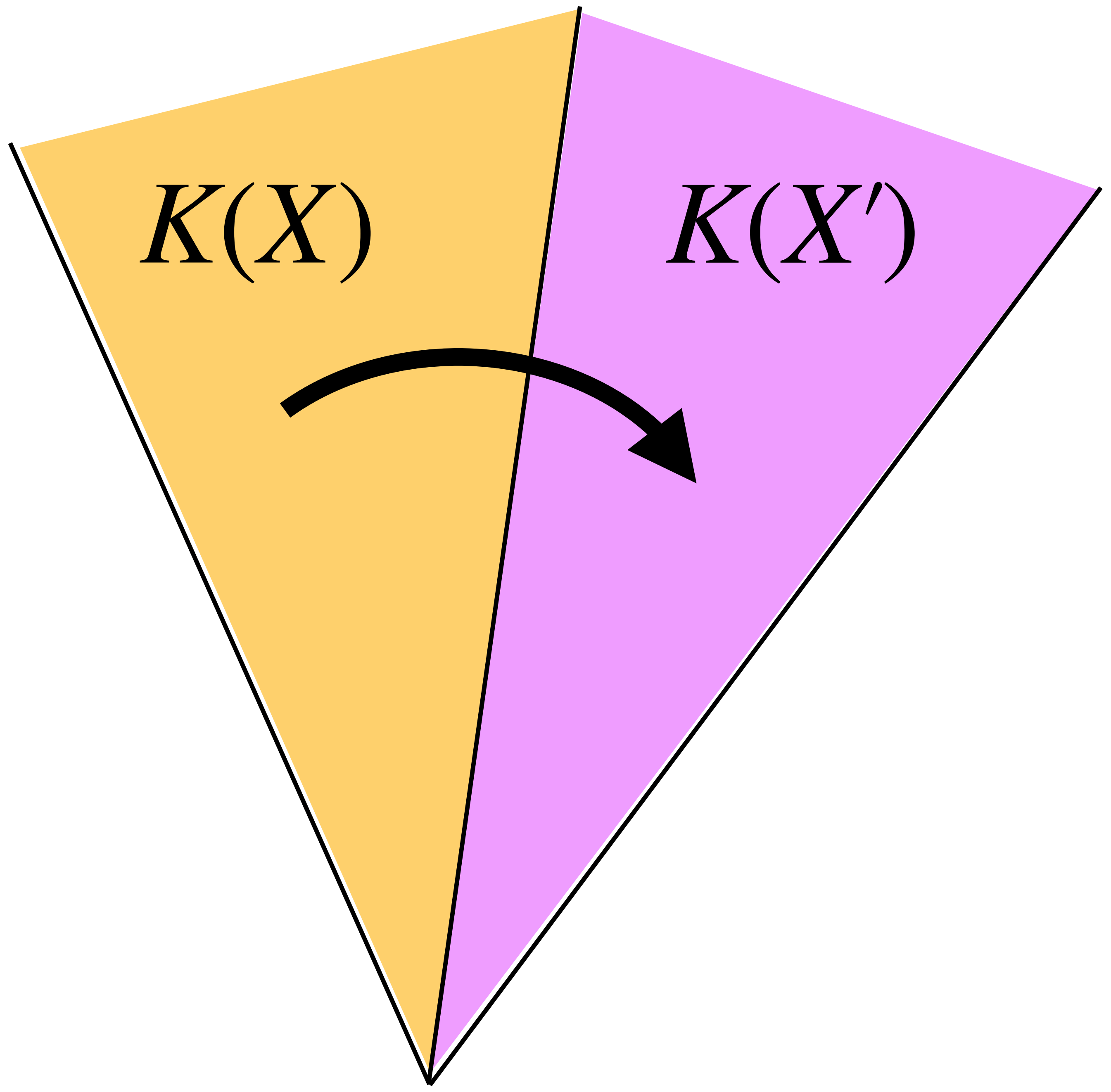
Symmetric flop: $X \cong X'$

[Brodie, Constantin, Lukas '20; Brodie, Constantin, Lukas, Ruehle '21]

In such a scenario, one can identify a **linear map** that maps divisors in X to divisors in X':

$$H_4(X, \mathbb{Z}) \to H_4(X, \mathbb{Z}) \,, \quad \vec{Q} \mapsto \Lambda \cdot \vec{Q}$$

Symmetric Flops in Calabi-Yaus



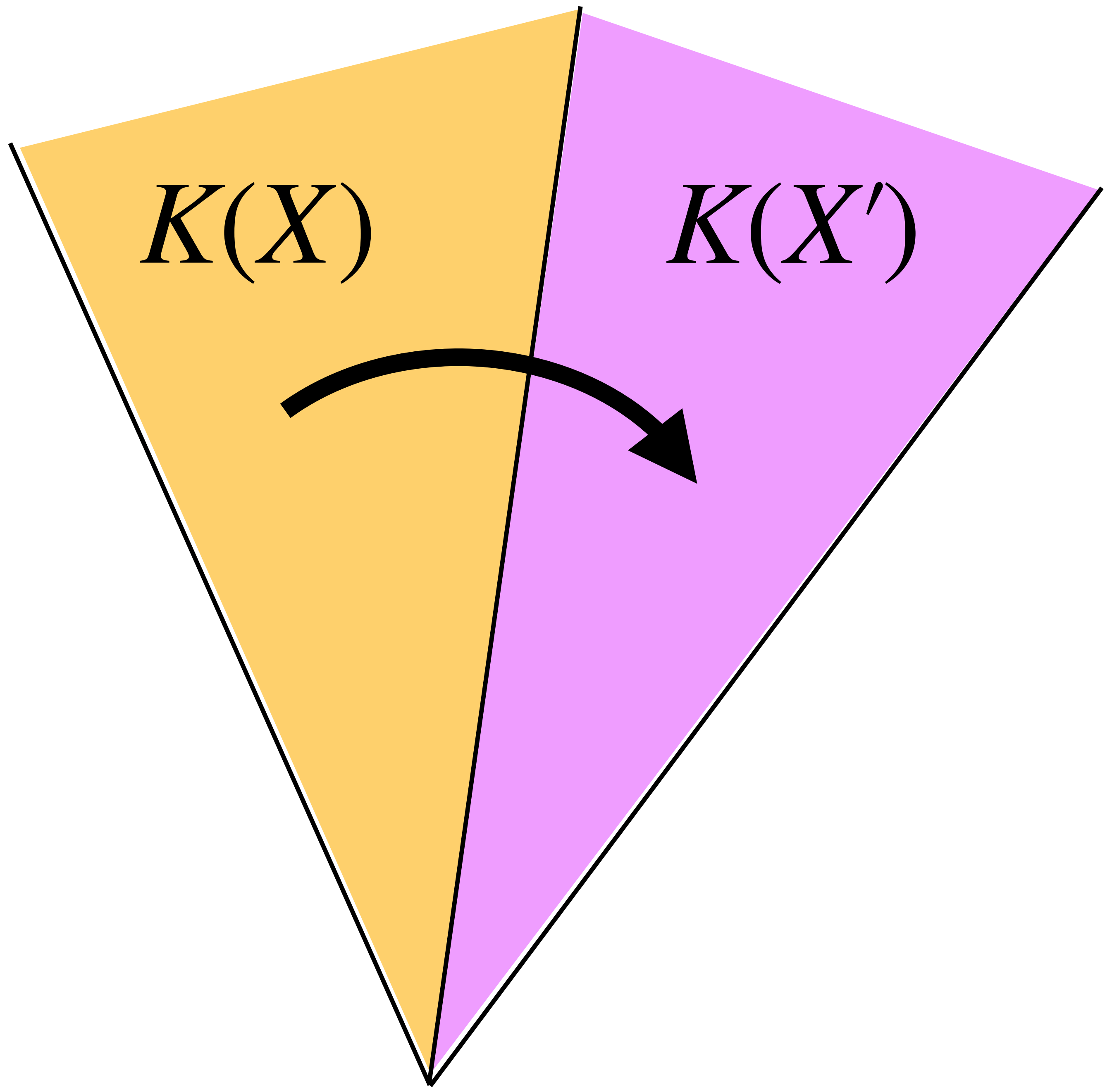
Symmetric flop: $X \cong X'$

[Brodie, Constantin, Lukas '20; Brodie, Constantin, Lukas, Ruehle '21]

In such a scenario, one can identify a **linear map** that maps divisors in X to divisors in X':

$$H_4(X, \mathbb{Z}) \to H_4(X, \mathbb{Z}) \, , \quad \boxed{\vec{Q} \mapsto \Lambda \cdot \vec{Q}}$$

Symmetric Flops in Calabi-Yaus



Symmetric flop: $X \cong X'$

[Brodie, Constantin, Lukas ’20; Brodie, Constantin, Lukas, Ruehle ’21]

In such a scenario, one can identify a **linear map** that maps divisors in X to divisors in X':

$$H_4(X, \mathbb{Z}) \to H_4(X, \mathbb{Z}) \,, \quad \boxed{\vec{Q} \mapsto \Lambda \cdot \vec{Q}}$$

Note: if Λ is of infinite order, then one can uncover an infinite number of independent effective divisors in this way.

Superpotential contributions

Superpotential contributions

Witten ['96] showed that a Euclidean D3-brane wrapping a **smooth** divisor in an orientifold of X contributes to the superpotential if

$$h^\bullet_+(D,\mathcal{O}_D) := \dim H^\bullet_+(D,\mathcal{O}_D) = (1,0,0)\,, \quad h^\bullet_-(D,\mathcal{O}_D) := \dim H^\bullet_-(D,\mathcal{O}_D) = 0$$

i.e. if the divisor is rigid.

Superpotential contributions

Witten ['96] showed that a Euclidean D3-brane wrapping a **smooth** divisor in an orientifold of X contributes to the superpotential if

$$h^\bullet_+(D,\mathcal{O}_D) := \dim H^\bullet_+(D,\mathcal{O}_D) = (1,0,0)\,, \quad h^\bullet_-(D,\mathcal{O}_D) := \dim H^\bullet_-(D,\mathcal{O}_D) = 0$$

i.e. if the divisor is rigid.

But **singular** divisors appear to be ubiquitous in Calabi-Yau threefolds.

When do singular divisors contribute to the superpotential?

Superpotential contributions

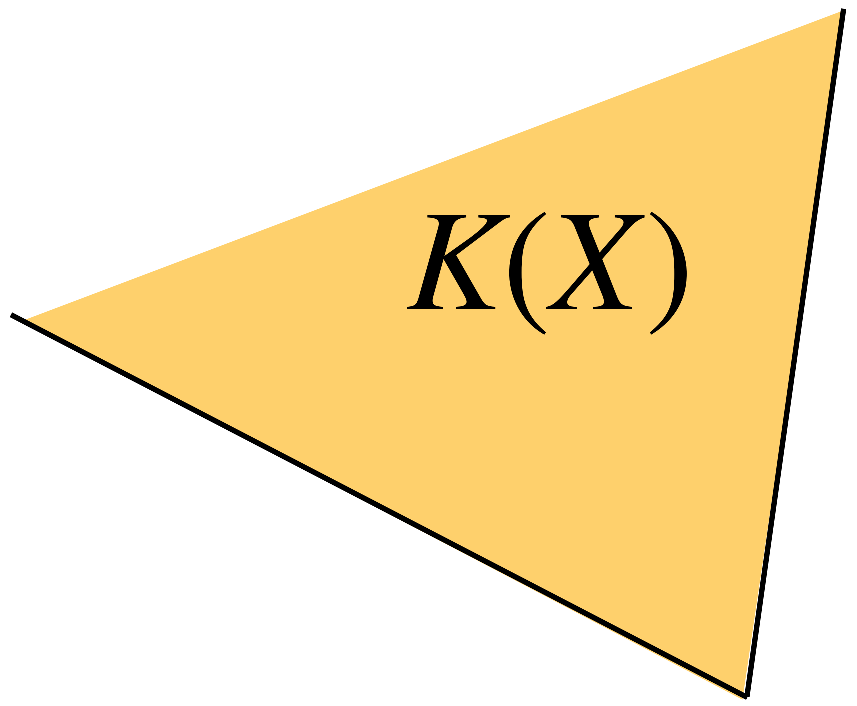
Superpotential contributions

Let us consider a superpotential that is generated by an ED3 on a smooth D in X:

Superpotential contributions

Let us consider a superpotential that is generated by an ED3 on a smooth D in X:

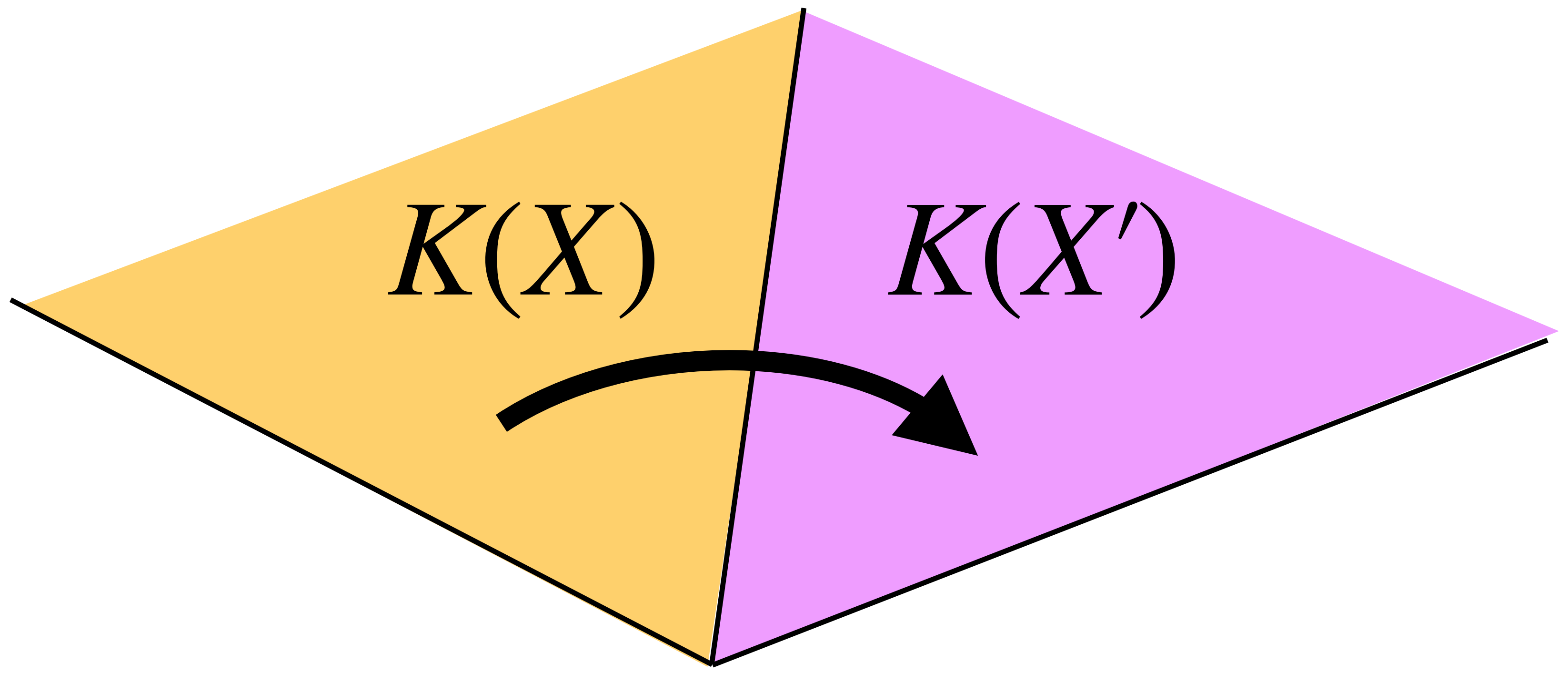
$$W_X \supset A_D\, e^{-2\pi T_D}$$



Superpotential contributions

Let us consider a superpotential that is generated by an ED3 on a smooth D in X:

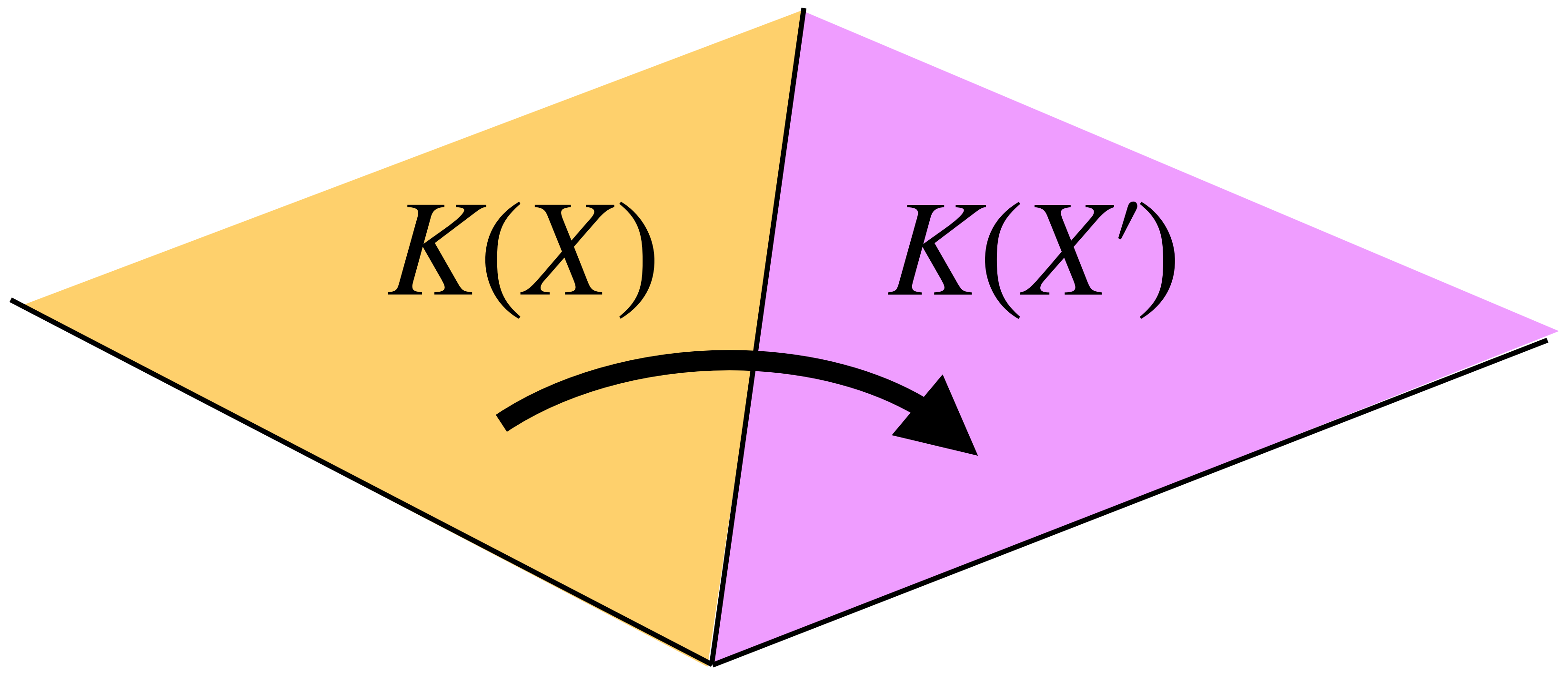
$W_X \supset A_D\, e^{-2\pi T_D}$



$W_{X'} \supset A_{D'}\, e^{-2\pi T_{D'}}$

Superpotential contributions

Let us consider a superpotential that is generated by an ED3 on a smooth D in X:

$W_X \supset A_D\, e^{-2\pi T_D}$  $W_{X'} \supset A_{D'}\, e^{-2\pi T_{D'}}$

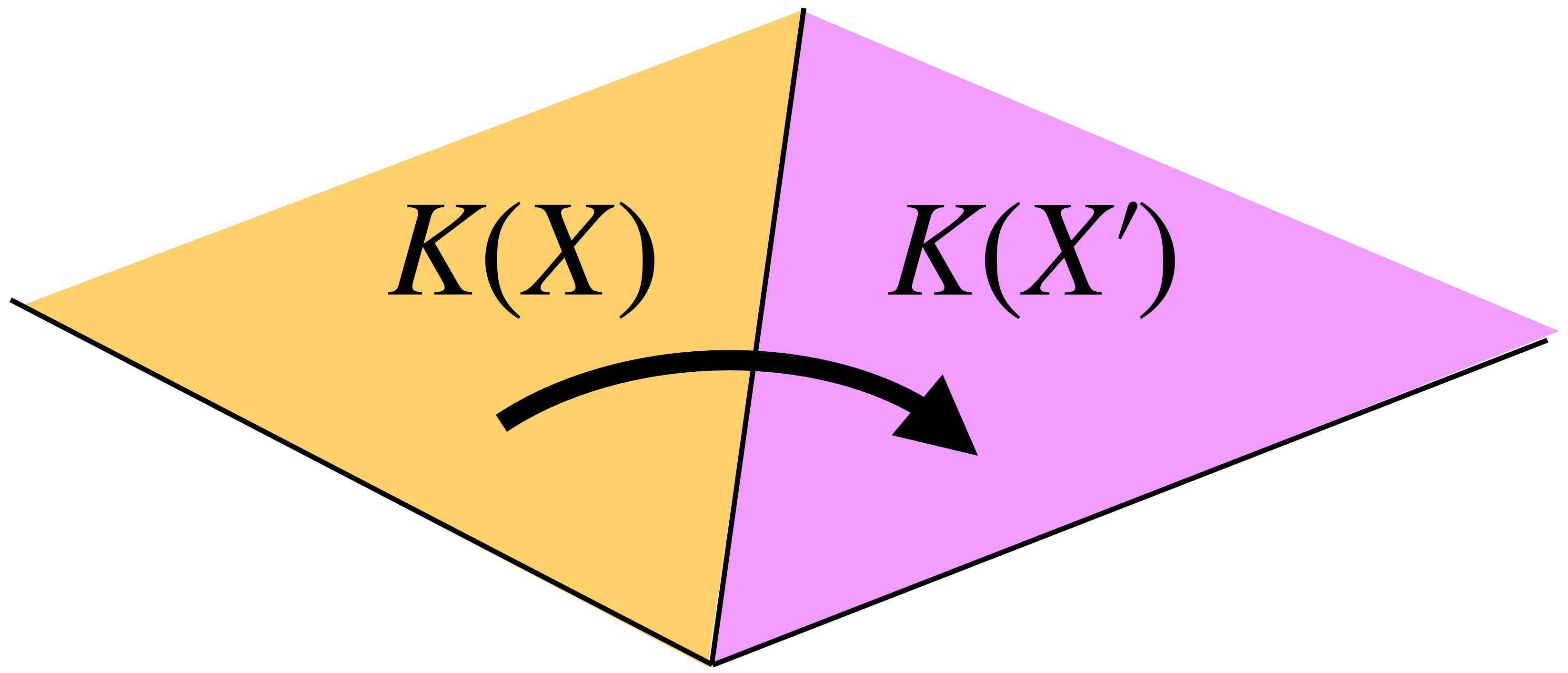
By holomorphy of the superpotential, we must have

$$W_X = W_{X'}$$

Superpotential contributions

Let us consider a superpotential that is generated by an ED3 on a smooth D in X:

$$W_X \supset A_D \, e^{-2\pi T_D}$$



$$W_{X'} \supset A_{D'} \, e^{-2\pi T_{D'}}$$

By holomorphy of the superpotential, we must have

$$W_X = W_{X'}$$

leads to a **new condition for a superpotential contribution:**

If a **singular divisor** can be flopped to a Calabi-Yau where it is **smooth and rigid**, then it contributes to the superpotential.

[NG, Kim, McAllister, Moritz, Stillman '22]

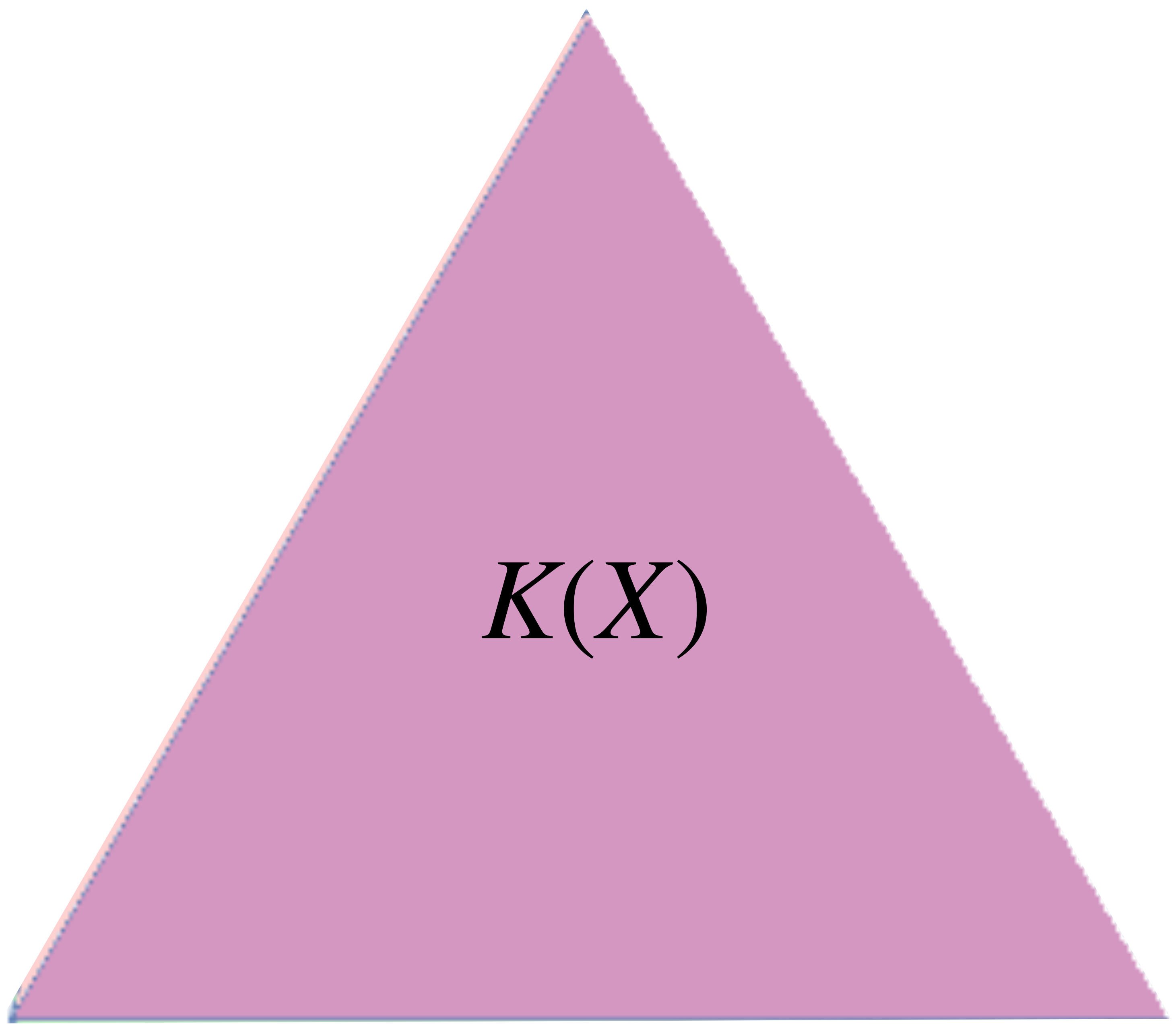
A modular superpotential

A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**

A modular superpotential

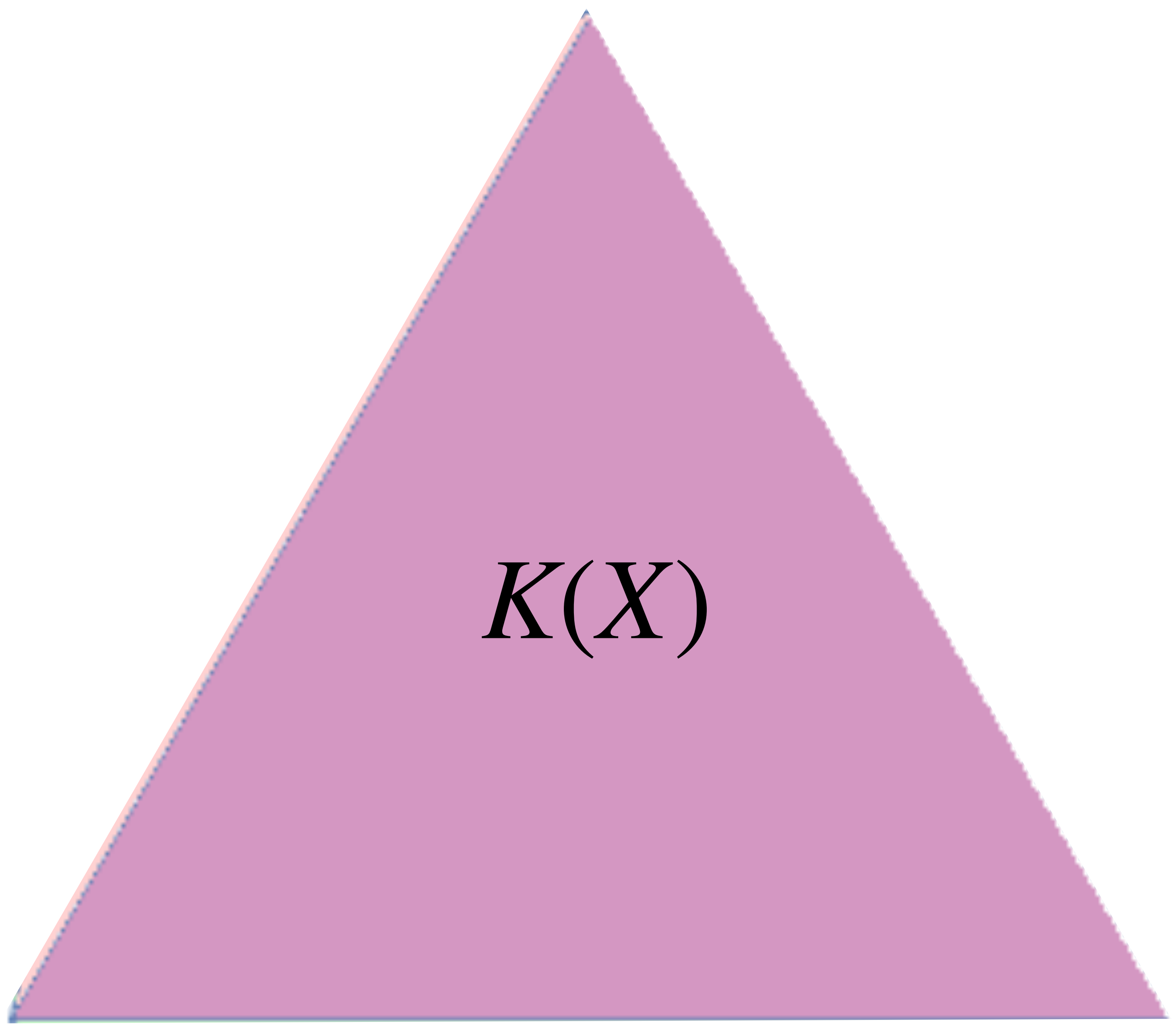
Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**



A 2D cross-section of the Kähler cone of X.

A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**



A 2D cross-section of the Kähler cone of X.

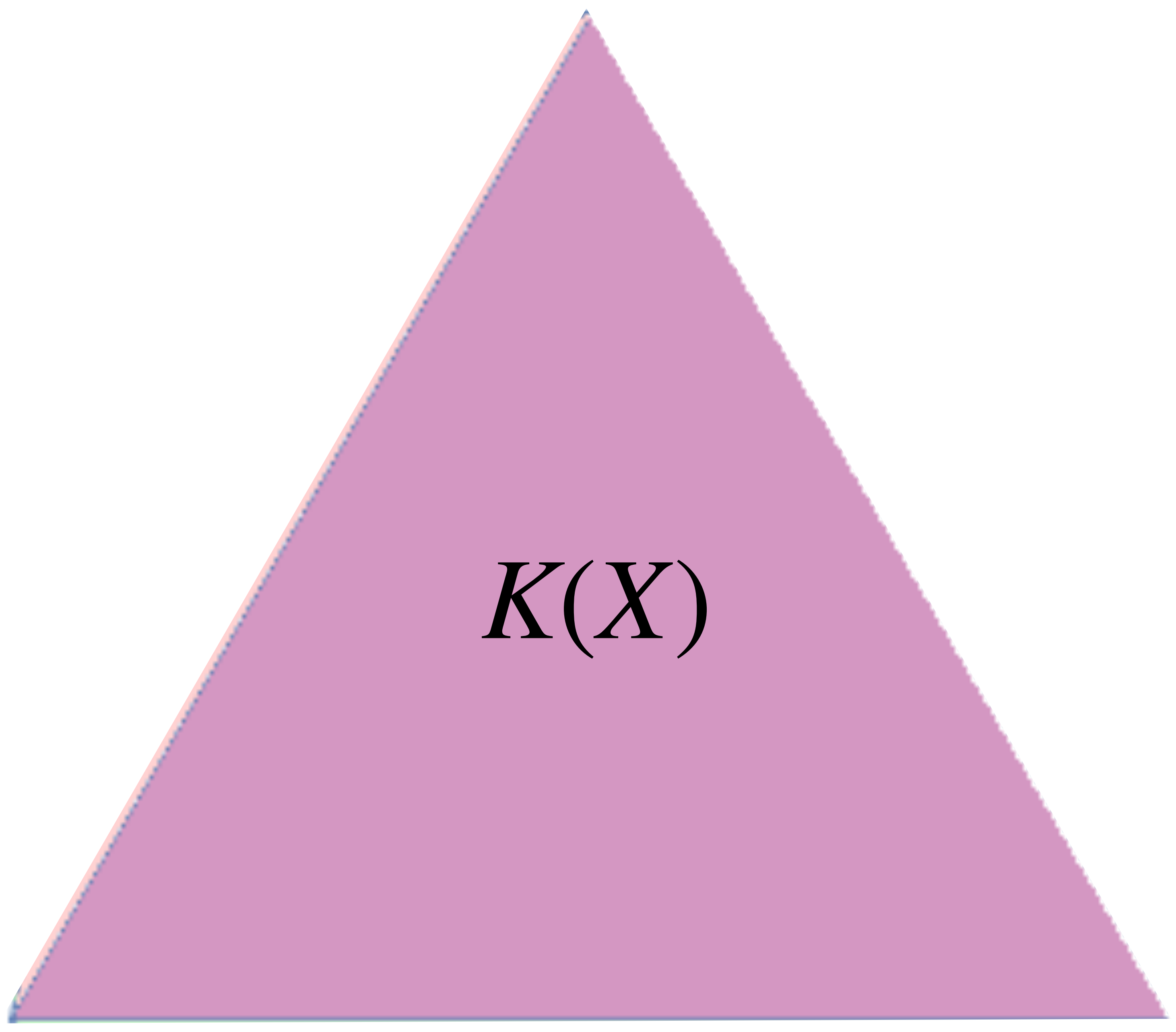
In this phase, we find a contribution to the superpotential from smooth, rigid divisors:

$$W = A_D \exp\left(-2\pi T_D\right)$$

[Witten '96]

A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**



A 2D cross-section of the Kähler cone of X.

In this phase, we find a contribution to the superpotential from smooth, rigid divisors:

$$W = A_D \exp\left(-2\pi T_D\right)$$ [Witten ’96]

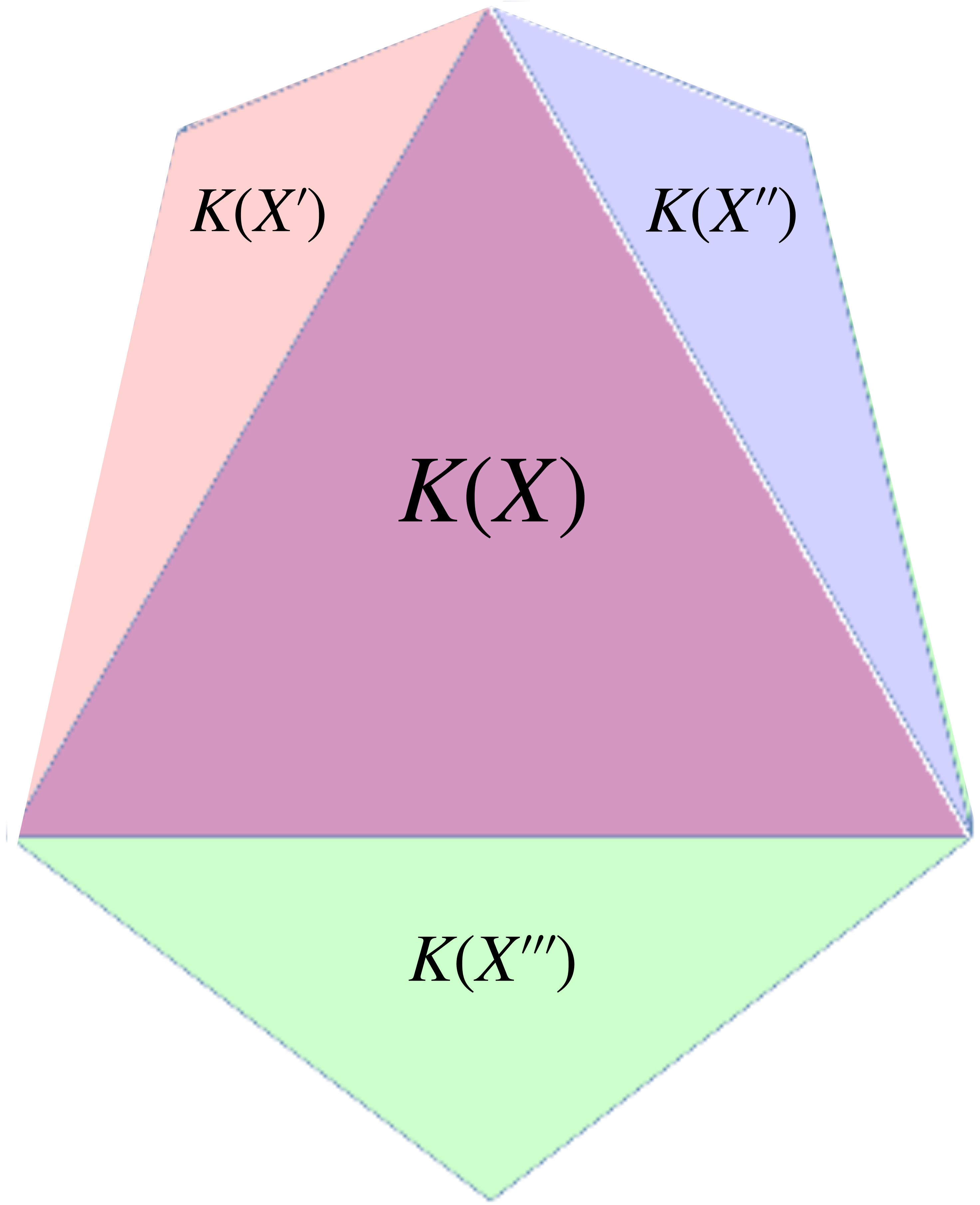
Now, let’s walk through each of the three walls of K(X) and see what happens.

A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**

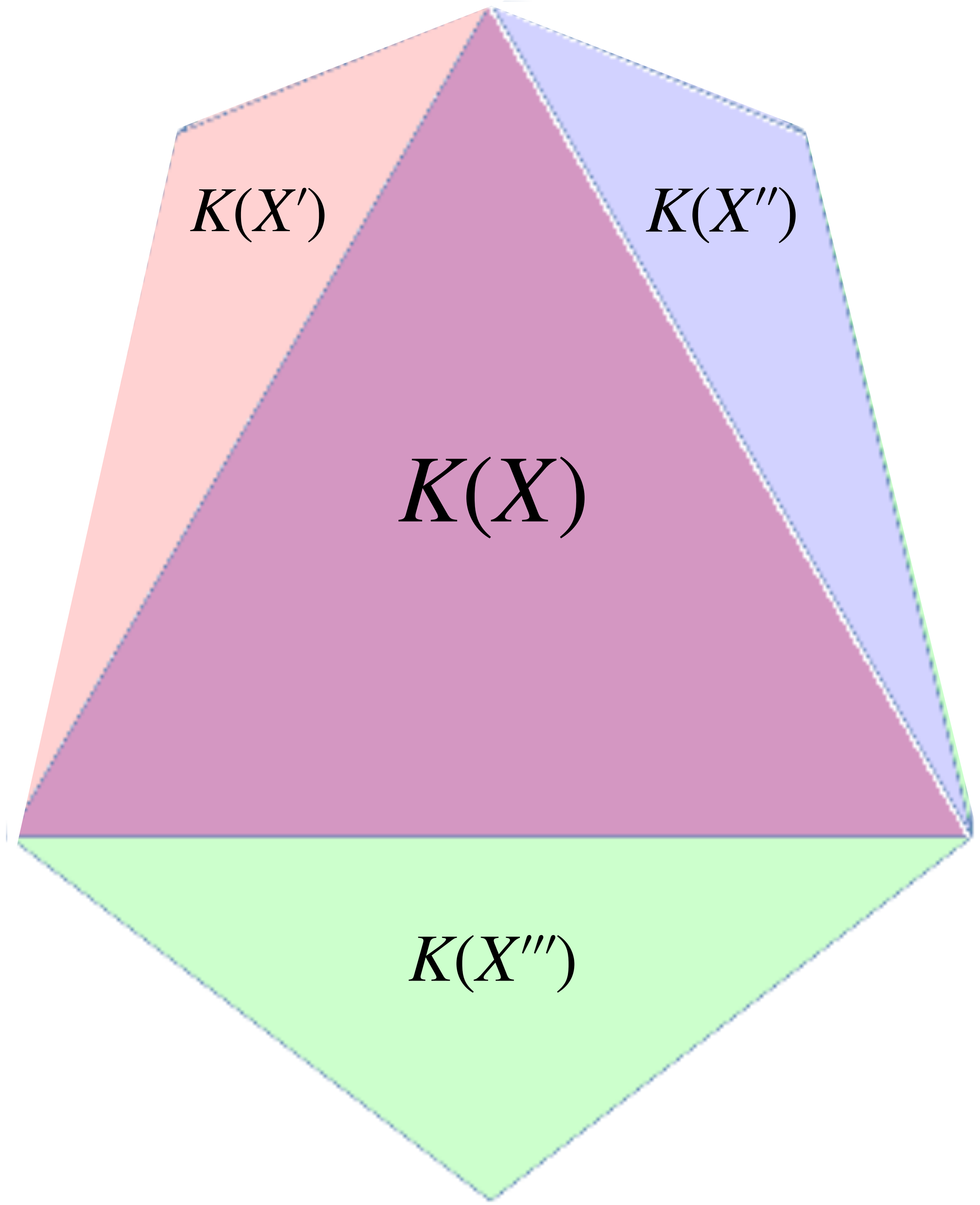
A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**



A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ.



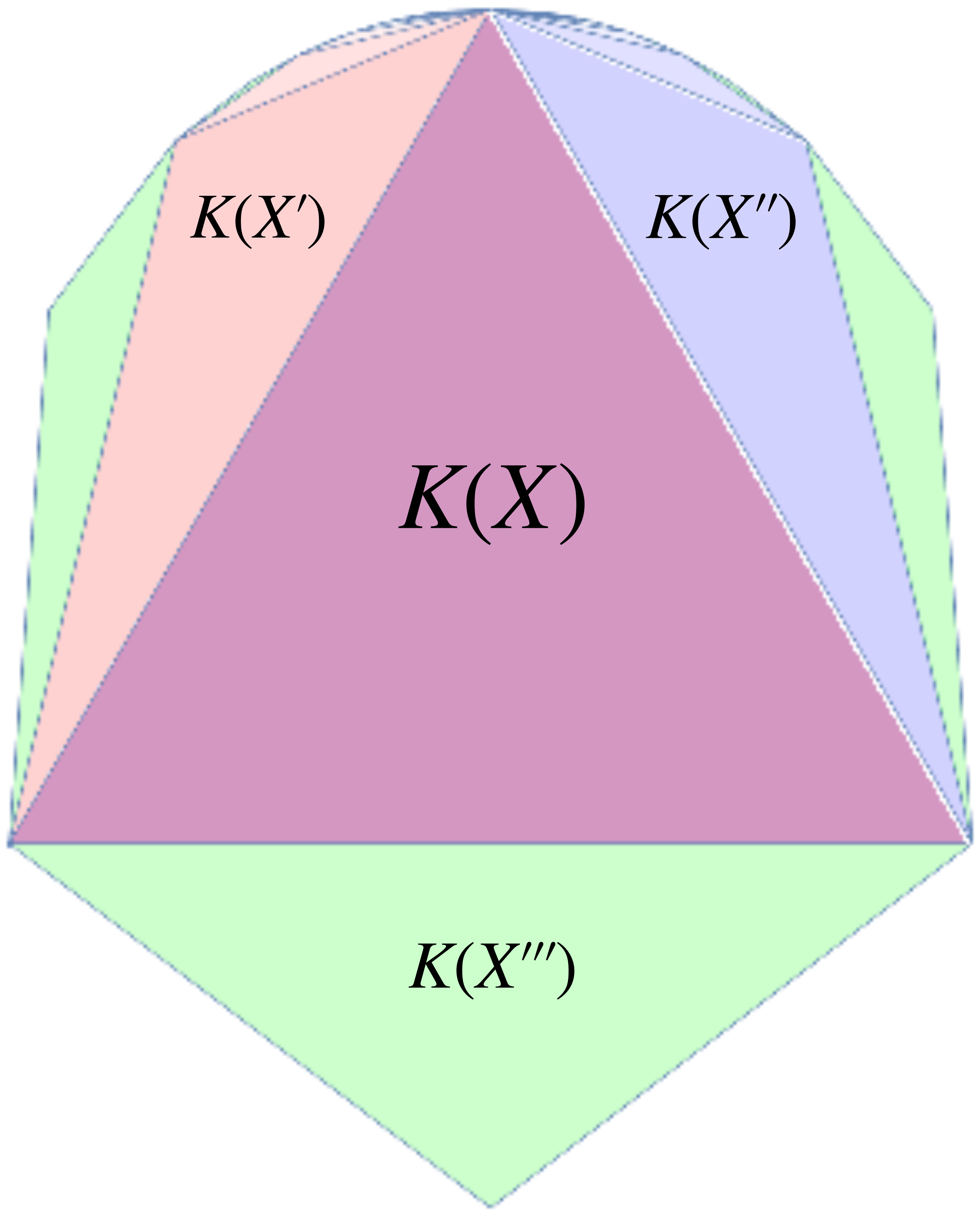
Analytically continuing through the symmetric flops, we get additional contributions from singular divisors:

$$W = A_D \exp\left(-2\pi T_D\right)$$

$$+A_D \exp\left(-2\pi \Lambda(T_D)\right)$$

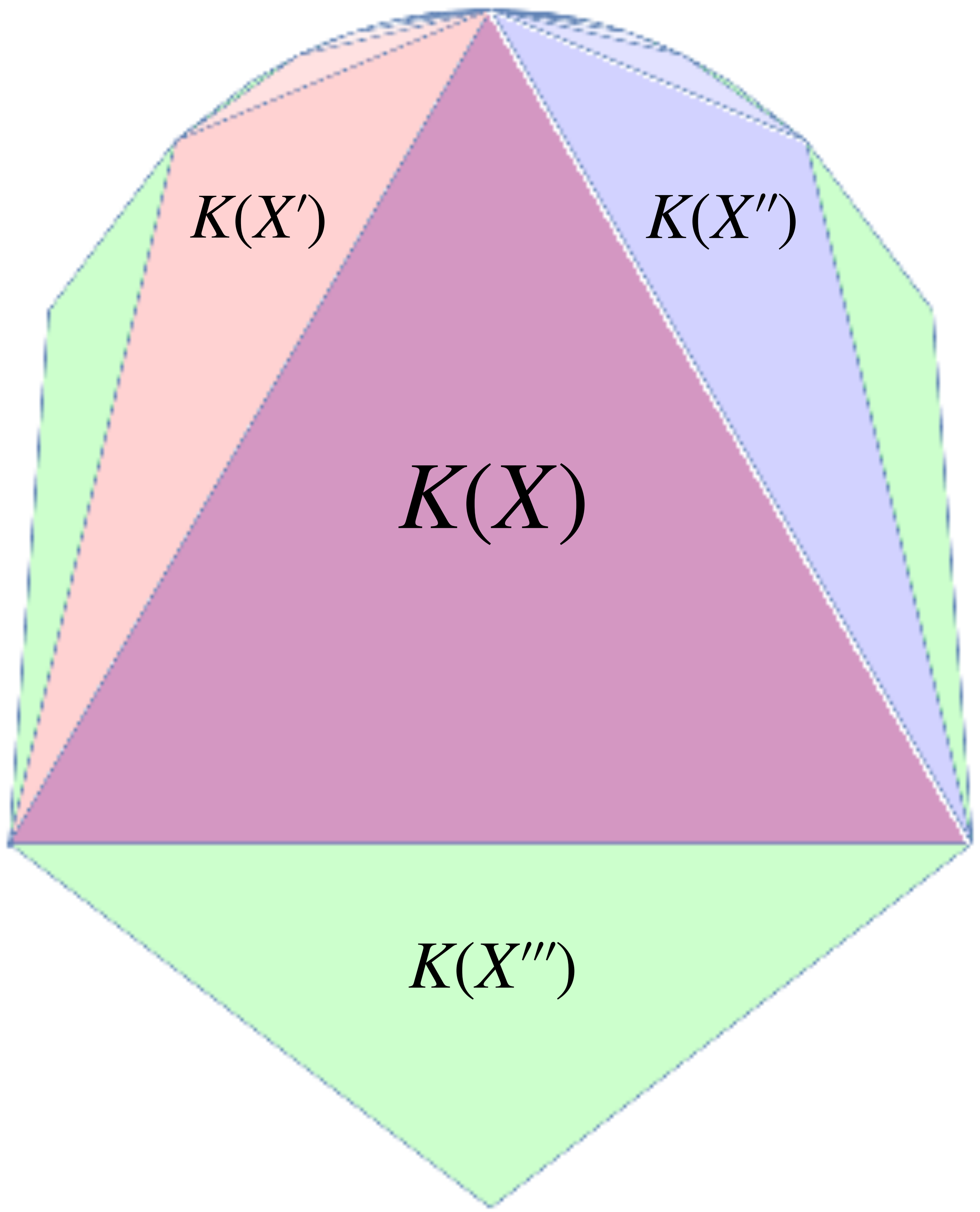
A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ**.**



A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ.

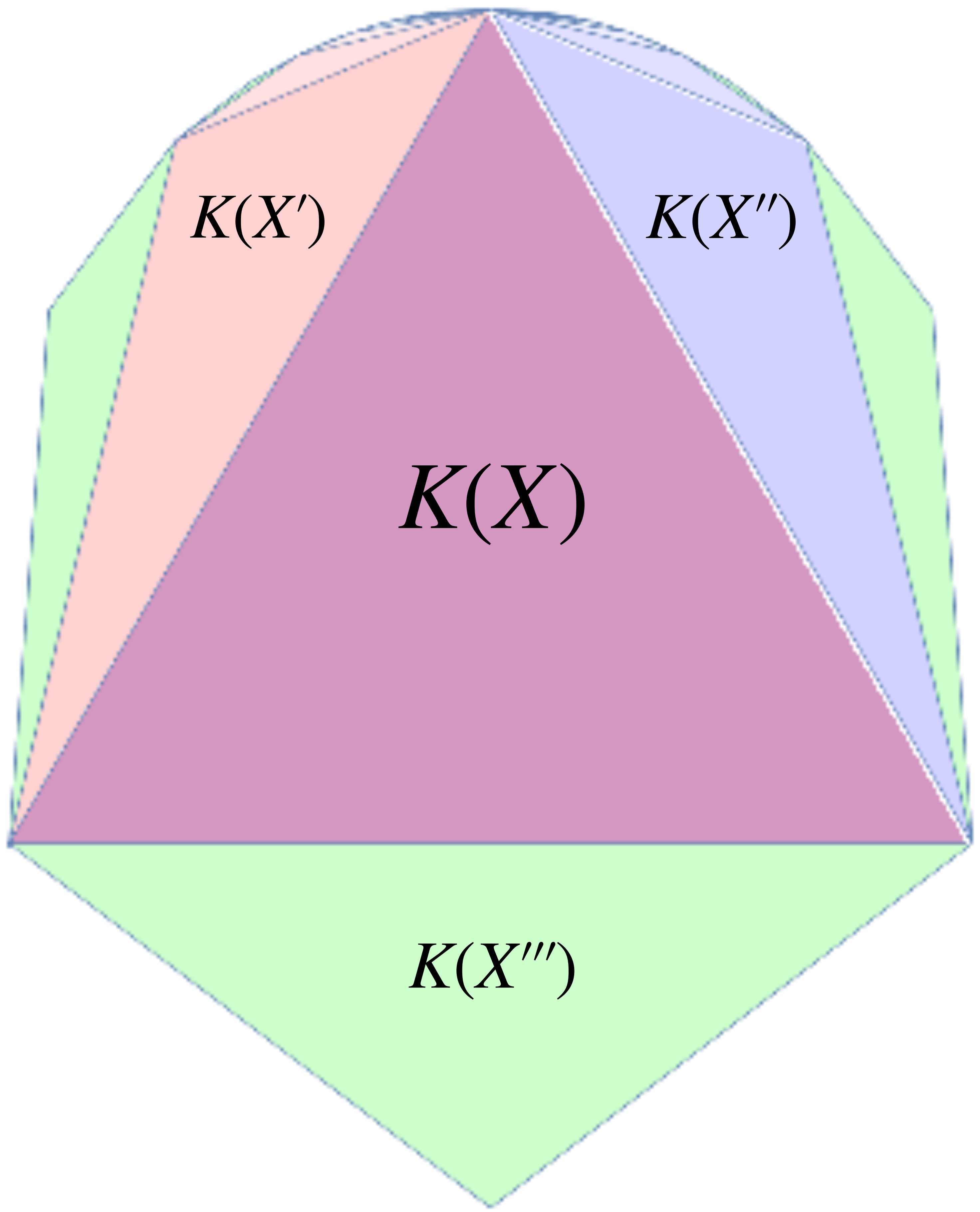


Analytically continuing through the **infinitely many** symmetric flops, we get:

$$W = A_D \exp\left(-2\pi T_D\right) + A_D \exp\left(-2\pi \Lambda(T_D)\right) + A_D \exp\left(-2\pi \Lambda^2(T_D)\right) + A_D \exp\left(-2\pi \Lambda^3(T_D)\right) + \ldots$$

A modular superpotential

Consider type IIB string theory on a particular Calabi-Yau threefold, X, with $h^{1,1} = 3$ that admits a **symmetric flop** induced by a linear map, Λ.



Analytically continuing through the **infinitely many** symmetric flops, we get:

$$
\begin{aligned}
W = & A_D \exp\left(-2\pi T_D\right) \\
& + A_D \exp\left(-2\pi \Lambda(T_D)\right) \\
& + A_D \exp\left(-2\pi \Lambda^2(T_D)\right) \\
& + A_D \exp\left(-2\pi \Lambda^3(T_D)\right) \\
& + \ \dots \\
& = f(T^i)\ \vartheta_{10}(z(T^i)\,;\tau(T^i))
\end{aligned}
$$

[c.f. Donagi, Grassi, Witten '96]

Conclusions

Conclusions

- Singular divisors appear ubiquitous in Calabi-Yau threefolds

Conclusions

- Singular divisors appear ubiquitous in Calabi-Yau threefolds
- We identified a sufficient condition for such singular divisors to contribute to the superpotential

Conclusions

- Singular divisors appear ubiquitous in Calabi-Yau threefolds
- We identified a sufficient condition for such singular divisors to contribute to the superpotential
- We used this condition to identify superpotentials that can be resummed into Jacobi theta functions

Conclusions

- Singular divisors appear ubiquitous in Calabi-Yau threefolds
- We identified a sufficient condition for such singular divisors to contribute to the superpotential
- We used this condition to identify superpotentials that can be resummed into Jacobi theta functions
- These modular superpotentials suggest the existence of strong-weak dualities involving the inversion of divisor volumes.

thank you!